Section 6.2
Antidifferentiation by Substitution

Definite Integral vs. Indefinite Integral

Definite Integral is a number, the limit of a sequence of Riemann Sums

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Indefinite Integral is a family of functions having a common derivative.

$$\int f(x)\,dx = F(x) + C$$

Example

$$\int x^2 - \sin x \, dx$$

General Antiderivative - Indefinite Integral

$$\int x^2 \, dx =$$

$$\int \frac{1}{x+1} \, dx =$$

$$\int \frac{1}{x^3} \, dx =$$

$$\int \cos(2x)\, dx =$$

$$\int e^{3x}\, dx =$$

$$\int x^2 + 7\, dx =$$

$$\int \sin(2x)\, dx =$$

$$\int e^{(1/3)x}\, dx =$$

Properties of Indefinite Integrals

$$\int k\,f(x)dx = k\int f(x)\,dx \quad \text{for any constant } k$$

$$\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

Power Formulas

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C \text{ when } n \neq -1$$

$$\int u^{-1} \, du = \int \frac{1}{u} \, du = \ln |u| + C$$

Trigonometric Formulas

$$\int \cos u \, du = \sin u + C$$

$$\int \sin u \, du = -\cos u + C$$

$$\int \sec^2 u \, du = \tan u + C$$

$$\int \csc^2 u \, du = -\cot u + C$$

$$\int \sec u \tan u \, du = \sec u + C$$

$$\int \csc u \cot u \, du = -\csc u + C$$

Exponential and Logarithmic Formulas

$$\int e^u \, du = e^u + C$$

$$\int a^u \, du = \frac{a^u}{\ln a} + C$$

$$\int \ln u \, du = u \ln u - u + C \quad \text{(See Example 2)}$$

$$\int \log_a u \, du = \int \frac{\ln u}{\ln a} \, du = \frac{u \ln u - u}{\ln a} + C$$

More Examples

$$\int u^{-1}\, du =$$

$$\int \ln u\, du =$$

Let $f(x) = x^3 + 1$ and let $u = x^2$. Find each of the following antiderivatives in terms of x:

a.) $\int f(x)\,dx =$

b.) $\int f(u)\,du =$

c.) $\int f(u)\,dx =$

$$\int \cos(x^3)\,dx =$$

Substitution in Indefinite Integrals

$$\int \cos(x^3) * x^2 \, dx =$$

$$\int \tan x \, dx =$$

$$\int 2x\sqrt{4 - x^2}\,dx =$$

$$\int \frac{4e^x}{\sqrt{e^x}}\,dx =$$

$$\int \sin x \, e^{\cos x} \, dx =$$

$$\int x^2 \sqrt{5 + 2x^3}\ dx =$$

$$\int \cot 7x \, dx = \int \frac{\cos 7x}{\sin 7x} dx$$

$$u = \sin 7x$$

$$\frac{du}{dx} = 7\cos 7x$$

$$\frac{du}{7} = \cos 7x \, dx$$

$$= \int \frac{1}{u} \cdot \frac{du}{7}$$

$$= \frac{1}{7} \int \frac{1}{u} du$$

$$= \frac{1}{7} \ln|u| + C$$

$$= \frac{1}{7} \ln|\sin 7x| + C$$

Find the indefinite integrals. In each case you can use a trigonometric identity to set up a substitution.

$$\int \frac{dx}{\cos^2 2x} = \int \sec^2 2x \, dx$$

$u = 2x$

$\frac{du}{dx} = 2$

$\frac{du}{2} = \frac{2dx}{2}$

$$= \frac{1}{2}\int \sec^2 u \, du$$

$$= \frac{1}{2}\tan u + C$$

$$= \frac{1}{2}\tan 2x + C$$

$$\int \cot^2 3x \, dx =$$

$$= \int (\csc^2 3x - 1)\, dx$$

$$= \frac{1}{3}\int (\csc^2 u - 1)\, du$$

$$= \frac{1}{3}(-\cot u - u) + C$$

$$= \frac{1}{3}(-\cot(3x) - 3x) + C$$

$$u = 3x$$

$$\frac{du}{dx} = 3$$

$$du = 3dx$$

$$\frac{du}{3} = dx$$

$$\int \cos^3 x \, dx = \int \cos^2 x \cos x \, dx$$

$$= \int (1 - \sin^2 x)(\cos x) \, dx$$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$$= \int (1 - u^2) \, du$$

$$= u - \frac{u^3}{3} + C$$

$$= \sin x - \frac{\sin^3 x}{3} + C$$

Substitution in Definite Integrals

Evaluate $\displaystyle\int_{0}^{\pi/3} \tan x \sec^2 x \, dx =$

$u = \tan x$

$\dfrac{du}{dx} = \sec^2 x$

$du = \sec^2 x \, dx$

$= \displaystyle\int_0^{\frac{\pi}{3}} u \, du$

$= \dfrac{u^2}{2}\Big|_0^{\frac{\pi}{3}}$

$= \dfrac{\tan^2 x}{2}\Big|_0^{\frac{\pi}{3}} = \dfrac{3}{2} - 0 = \dfrac{3}{2}$

$=$

$$\int_0^{\pi} e^{\sin x} \cos x \, dx = \int_0^{\pi} e^u \, du$$

$$= e^u \Big|_0^{\pi}$$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$$= e^{\sin x} \Big|_0^{\pi}$$

$$= e^{\sin \pi} - e^{\sin 0}$$

$$= 1 - 1 = 0$$

Evaluate $\displaystyle\int \frac{t}{\sqrt{2t^2 + 1}}\, dt =$

$$\int \frac{\ln x}{x}\, dx =$$

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt =$$

$$\int_2^5 \frac{\sqrt{\ln x}}{x}\, dx =$$

Evaluate $$\int_0^1 \frac{x}{x^2 - 4}\, dx =$$